**Sub-question 1.a: Distinguish between numerical and analytical methods in the solution of mathematical problems.**

1. *Easy:* Can you define what a numerical method is?
2. *Intermediate:* Can you give an example of a problem that would be solved using a numerical method?
3. *Intermediate:* Can you define what an analytical method is?
4. *Advanced:* Can you give an example of a problem that would be solved using an analytical method?
5. *Expert:* Can you explain a situation where you would prefer to use a numerical method over an analytical method, and vice versa?

**Sub-question 1.b: Describe the bisection method and explain how it differs from the Newton-Raphson method.**

1. *Easy:* Can you explain what the bisection method is?
2. *Intermediate:* Can you describe a situation where you would use the bisection method?
3. *Intermediate:* Can you explain what the Newton-Raphson method is?
4. *Advanced:* Can you describe a situation where you would use the Newton-Raphson method?
5. *Expert:* Can you compare and contrast the bisection method and the Newton-Raphson method in terms of efficiency, speed, and reliability?

**Sub-question 1.c: Given an initial guess x=3, find an approximate value of the root of the function f(x) = x^2 using 3 iterations. Provide a sample R code that you would use to solve this problem, stopping after 40 iterations.**

1. *Easy:* Can you explain what an iteration is in the context of numerical methods?
2. *Intermediate:* Can you describe how you would use iterations to find an approximate value of a root of a function?
3. *Intermediate:* Can you write a simple R code to perform iterations?
4. *Advanced:* Can you modify the R code to stop after a certain number of iterations?
5. *Expert:* Can you write an R code to find an approximate value of the root of a specific function using a specific method, such as the Newton-Raphson method, and stop after a certain number of iterations?

**Sub-question 1.d: Using the inverse-transform approach to explain (mathematically) how you would generate random numbers from the exponential distribution, f(x) = exp(-x), and further provide an R code that will be used to generate random numbers from this distribution.**

1. *Easy:* Can you explain what the inverse-transform method is?
2. *Intermediate:* Can you describe how you would use the inverse-transform method to generate random numbers from a distribution?
3. *Intermediate:* Can you write a simple R code to generate random numbers from a distribution?
4. *Advanced:* Can you modify the R code to generate random numbers from the exponential distribution?
5. *Expert:* Can you explain the mathematical reasoning behind using the inverse-transform method to generate random numbers from the exponential distribution, and demonstrate this with an R code?

*Question 2: Starting with the Newton-Raphson formula, shown that the order of convergence of the Newton-Raphson method is |e\_n+1| ≤ C|e\_n|^2 as n → ∞, with C = |f''(x)/2f'(x*)| provided |e\_0| ≤ ε.\*\*

1. *Easy:* Can you explain what the Newton-Raphson formula is used for?
2. *Intermediate:* What is the concept of convergence in numerical methods?
3. *Intermediate:* Can you explain the term "order of convergence" in the context of the Newton-Raphson method?
4. *Advanced:* How would you prove that the order of convergence of the Newton-Raphson method is quadratic?
5. *Expert:* Can you derive the condition under which the Newton-Raphson method converges quadratically?

**Mixed Topics and Concepts:**

1. *Easy:* Can you explain what a root-finding algorithm is?
2. *Intermediate:* Can you compare and contrast the Newton-Raphson method with other root-finding methods like the bisection method and the secant method?
3. *Intermediate:* Can you explain the concept of error analysis in numerical methods?
4. *Advanced:* How would you apply the concept of error analysis to the Newton-Raphson method?
5. *Expert:* Can you explain how the speed of convergence of a root-finding method affects its efficiency and reliability?

**Question 3: Use quadratic spline interpolation to find the approximate value of y at x=3. Use polynomial interpolation to determine the value of the function at x=2.7.**

1. *Easy:* Can you explain what interpolation is?
2. *Intermediate:* What is the difference between quadratic spline interpolation and polynomial interpolation?
3. *Intermediate:* How would you use quadratic spline interpolation to estimate a value?
4. *Advanced:* How would you use polynomial interpolation to estimate a value?
5. *Expert:* Can you explain the mathematical reasoning behind using quadratic spline interpolation and polynomial interpolation to estimate values?

**Mixed Topics and Concepts:**

1. *Easy:* Can you explain what a spline is?
2. *Intermediate:* Can you compare and contrast different types of interpolation methods, such as linear interpolation, polynomial interpolation, and spline interpolation?
3. *Intermediate:* Can you explain the concept of curve fitting in numerical methods?
4. *Advanced:* How would you apply the concept of curve fitting to interpolation methods?
5. *Expert:* Can you explain how the choice of interpolation method affects the accuracy and reliability of the estimated values?

**Question 4: Consider the general linear model. Using a maximum likelihood approach, clearly showing the likelihood function, the log-likelihood function, and the score-vector, derive the maximum likelihood estimators of β and σ².**

1. *Easy:* Can you explain what a general linear model is?
2. *Intermediate:* What is the concept of maximum likelihood estimation?
3. *Intermediate:* How would you derive the maximum likelihood estimators of parameters in a general linear model?
4. *Advanced:* Can you explain the likelihood function, the log-likelihood function, and the score-vector in the context of maximum likelihood estimation?
5. *Expert:* Can you derive the maximum likelihood estimators of parameters in a general linear model using a maximum likelihood approach?

**Mixed Topics and Concepts:**

1. *Easy:* Can you explain what a statistical model is?
2. *Intermediate:* Can you compare and contrast different types of statistical models, such as linear models, generalized linear models, and nonlinear models?
3. *Intermediate:* Can you explain the concept of parameter estimation in statistical models?
4. *Advanced:* How would you apply the concept of maximum likelihood estimation to different types of statistical models?
5. *Expert:* Can you explain how the choice of statistical model and estimation method affects the accuracy and reliability of the estimated parameters?

**Sub-question 1.a: Distinguish between numerical and analytical methods in the solution of mathematical problems.**

1. *Easy:* Can you define what a numerical method is?
2. *Easy:* Can you define what an analytical method is?
3. *Intermediate:* Can you give an example of a problem that would be solved using a numerical method?
4. *Intermediate:* Can you give an example of a problem that would be solved using an analytical method?
5. *Intermediate:* Can you explain the concept of approximation in numerical methods?
6. *Intermediate:* Can you explain the concept of exact solutions in analytical methods?
7. *Advanced:* Can you explain a situation where you would prefer to use a numerical method over an analytical method, and vice versa?
8. *Advanced:* Can you discuss the limitations of numerical methods?
9. *Advanced:* Can you discuss the limitations of analytical methods?
10. *Expert:* Can you explain how the choice between numerical and analytical methods affects the accuracy and reliability of the solution?

**Sub-question 1.b: Describe the bisection method and explain how it differs from the Newton-Raphson method.**

1. *Easy:* Can you explain what the bisection method is?
2. *Easy:* Can you explain what the Newton-Raphson method is?
3. *Intermediate:* Can you describe a situation where you would use the bisection method?
4. *Intermediate:* Can you describe a situation where you would use the Newton-Raphson method?
5. *Intermediate:* Can you explain the concept of root-finding in numerical methods?
6. *Intermediate:* Can you explain the concept of iteration in numerical methods?
7. *Advanced:* Can you compare and contrast the bisection method and the Newton-Raphson method in terms of efficiency, speed, and reliability?
8. *Advanced:* Can you discuss the limitations of the bisection method?
9. *Advanced:* Can you discuss the limitations of the Newton-Raphson method?
10. *Expert:* Can you explain how the choice between the bisection method and the Newton-Raphson method affects the accuracy and reliability of the root-finding process?

**Sub-question 1.c: Given an initial guess x=3, find an approximate value of the root of the function f(x) = x^2 using 3 iterations. Provide a sample R code that you would use to solve this problem, stopping after 40 iterations.**

1. *Easy:* Can you explain what an iteration is in the context of numerical methods?
2. *Easy:* Can you explain what a root of a function is?
3. *Intermediate:* Can you describe how you would use iterations to find an approximate value of a root of a function?
4. *Intermediate:* Can you write a simple R code to perform iterations?
5. *Intermediate:* Can you explain the concept of stopping criteria in numerical methods?
6. *Intermediate:* Can you explain the concept of initial guess in root-finding methods?
7. *Advanced:* How would you modify the R code to stop after a certain number of iterations?
8. *Advanced:* Can you discuss the effect of the choice of initial guess on the root-finding process?
9. *Advanced:* Can you discuss the effect of the number of iterations on the accuracy of the root-finding process?
10. *Expert:* Can you write an R code to find an approximate value of the root of a specific function using a specific method, such as the Newton-Raphson method, andstop after a certain number of iterations?

**Sub-question 1.d: Using the inverse-transform approach to explain (mathematically) how you would generate random numbers from the exponential distribution, f(x) = exp(-x), and further provide an R code that will be used to generate random numbers from this distribution.**

1. *Easy:* Can you explain what the inverse-transform method is?
2. *Easy:* Can you explain what the exponential distribution is?
3. *Intermediate:* Can you describe how you would use the inverse-transform method to generate random numbers from a distribution?
4. *Intermediate:* Can you write a simple R code to generate random numbers from a distribution?
5. *Intermediate:* Can you explain the concept of random number generation in statistical simulations?
6. *Intermediate:* Can you explain the concept of probability density function in the context of random number generation?
7. *Advanced:* How would you modify the R code to generate random numbers from the exponential distribution?
8. *Advanced:* Can you discuss the effect of the choice of distribution on the properties of the generated random numbers?
9. *Advanced:* Can you discuss the limitations of the inverse-transform method for generating random numbers?
10. *Expert:* Can you explain the mathematical reasoning behind using the inverse-transform method to generate random numbers from the exponential distribution, and demonstrate this with an R code?

**Sub-question 1.d: Using the inverse-transform approach to explain (mathematically) how you would generate random numbers from the exponential distribution, f(x) = exp(-x), and further provide an R code that will be used to generate random numbers from this distribution.**

1. *Easy:* Can you explain what the inverse-transform method is?
2. *Easy:* Can you explain what the exponential distribution is?
3. *Intermediate:* Can you describe how you would use the inverse-transform method to generate random numbers from a distribution?
4. *Intermediate:* Can you write a simple R code to generate random numbers from a distribution?
5. *Intermediate:* Can you explain the concept of random number generation in statistical simulations?
6. *Intermediate:* Can you explain the concept of probability density function in the context of random number generation?
7. *Advanced:* How would you modify the R code to generate random numbers from the exponential distribution?
8. *Advanced:* Can you discuss the effect of the choice of distribution on the properties of the generated random numbers?
9. *Advanced:* Can you discuss the limitations of the inverse-transform method for generating random numbers?
10. *Expert:* Can you explain the mathematical reasoning behind using the inverse-transform method to generate random numbers from the exponential distribution, and demonstrate this with an R code?

**Mixed Topics and Concepts:**

1. *Easy:* Can you explain what a probability distribution is?
2. *Easy:* Can you explain what a random variable is?
3. *Intermediate:* Can you compare and contrast different types of probability distributions, such as the exponential distribution, the normal distribution, and the uniform distribution?
4. *Intermediate:* Can you explain the concept of simulation in statistical analysis?
5. *Intermediate:* Can you explain the concept of transformation of random variables?
6. *Advanced:* How would you apply the concept of transformation to generate random numbers from a specific distribution?
7. *Advanced:* Can you discuss the effect of the choice of distribution on the properties of the generated random numbers?
8. *Advanced:* Can you discuss the limitations of using simulation to generate random numbers?
9. *Expert:* Can you explain how the choice of method for generating random numbers affects the accuracy and reliability of the statistical analysis?
10. *Expert:* Can you explain how the properties of the probability distribution affect the properties of the generated random numbers?

**Numerical and Analytical Methods:**

1. *Easy:* Define the terms "numerical method" and "analytical method" in the context of mathematical problem-solving.
2. *Intermediate:* Provide an example of a mathematical problem that can be solved using both numerical and analytical methods. Compare the solutions obtained by both methods.
3. *Advanced:* Discuss the advantages and disadvantages of using numerical methods over analytical methods in solving mathematical problems.
4. *Expert:* Given a complex mathematical problem, describe how you would decide whether to use a numerical or analytical method to solve it.

**Bisection and Newton-Raphson Methods:**

1. *Easy:* Define the bisection method and the Newton-Raphson method.
2. *Intermediate:* Provide an example of a mathematical problem that can be solved using both the bisection method and the Newton-Raphson method. Compare the solutions obtained by both methods.
3. *Advanced:* Discuss the advantages and disadvantages of using the bisection method over the Newton-Raphson method in finding the root of a function.
4. *Expert:* Given a complex root-finding problem, describe how you would decide whether to use the bisection method or the Newton-Raphson method to solve it.

**Inverse-Transform Method and Exponential Distribution:**

1. *Easy:* Define the inverse-transform method and the exponential distribution.
2. *Intermediate:* Provide an example of how the inverse-transform method can be used to generate random numbers from the exponential distribution.
3. *Advanced:* Discuss the advantages and disadvantages of using the inverse-transform method over other methods in generating random numbers from the exponential distribution.
4. *Expert:* Given a complex statistical simulation problem, describe how you would decide whether to use the inverse-transform method or another method to generate random numbers from a specific distribution.

**Root Finding and Iteration:**

1. *Easy:* Define the term "root" in the context of a mathematical function.
2. *Intermediate:* Provide an example of a mathematical problem that involves finding the root of a function. How would you solve it?
3. *Advanced:* Discuss the concept of iteration in the context of root finding. How does the number of iterations affect the accuracy of the solution?
4. *Expert:* Given a complex root-finding problem, describe how you would decide on the number of iterations to use. Discuss the trade-off between accuracy and computational cost.

**Random Number Generation and Simulation:**

1. *Easy:* Define the term "random number generation" in the context of statistical simulations.
2. *Intermediate:* Provide an example of a statistical simulation that requires random number generation. How would you carry it out?
3. *Advanced:* Discuss the concept of probability distribution in the context of random number generation. How does the choice of distribution affect the properties of the generated random numbers?
4. *Expert:* Given a complex statistical simulation problem, describe how you would decide on the method and distribution to use for random number generation. Discuss the trade-off between accuracy and computational cost.

**R Programming and Code Writing:**

1. *Easy:* Define the term "R programming" in the context of statistical computing.
2. *Intermediate:* Provide an example of a mathematical problem that can be solved using R programming. How would you write the code?
3. *Advanced:* Discuss the concept of code efficiency in the context of R programming. How does the way you write your code affect the speed and reliability of your solution?
4. *Expert:* Given a complex mathematical problem, describe how you would decide on the structure and syntax to use in your R code. Discuss the trade-off between code readability and computational efficiency.